

9. WindPRO – MODELE CFD-Interface

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Les auteurs de cette section sont :

Morten Lybech Thøgersen, Mads V. Sørensen et Per Nielsen, EMD International A/S, Aalborg,

9.0 Introduction et guide

9.0.0 Introduction

Quand le relief du terrain est très complexe l'étude des gisements éoliens ne peut pas être faite avec les modèles traditionnels qui ont montré leurs limites dans ces conditions. La solution la plus radicale serait d'incorporer dans WindPRO un modèle numérique d'écoulement (CFD : Computational Fluid Dynamics, ce terme sera utilisé dans la suite de ce document).

Plusieurs logiciels utilisant un modèle CFD et permettant l'étude du vent sont commercialisés. CFD-Interface a pour objet de faire l'interface entre WindPRO et ces logiciels.

CFD-Interface a deux fonctions :

- Le *Pré-traitement* qui génère les fichiers destinés aux logiciels CFD à partir des données présentes dans WindPRO.
- Le *Post-traitement* qui compare les résultats produits par un logiciel CFD avec ceux produits par WindPRO avec WAsP.

Note : les cartes de ressources éoliennes produites par les logiciels CFD peuvent être utilisées directement pour les calculs avec PARK.

9.0.1 Guide des étapes à suivre

Les principales étapes à suivre sont les suivantes :

- Entrer dans WindPRO l'orographie et les rugosités de la zone du projet (voir BASIS pour plus d'informations).
- Créer un objet *Données-site* (ayant pour usage *Calcul de cartes de ressources...*) pointant sur les objets ou les fichiers contenant les données d'orographie et de rugosité.
- Eventuellement importer les données de vent dans un objet *Meteo*.
- Lancer le calcul CFD-Interface, choisir *Pré-traitement*, sélectionner les données et le système de coordonnées qui seront utilisés par le logiciel CFD.
- Choisir le dossier où seront enregistrés les fichiers de données. Les fichiers sont générés au format WAsP car lisible par la plupart des logiciels CFD.
- Calculer la carte des ressources avec le logiciel CFD ; le fichier carte des ressources produit devra être au format .rsf ou .wrg (WAsP).
- Lancer le calcul CFD-Interface, choisir *Post-traitement*, charger la carte des ressources produite par le logiciel CFD, charger la même carte des ressources produite avec WindPRO et WAsP.
- Comparer les résultats, différentes grandeurs peuvent comparées.

Calcul du productible annuel avec PARK :

- Démarrer un calcul PARK, dans l'onglet *Données de vent* choisir *Utiliser les données d'un fichier ressources*, sélectionner le fichier carte des ressources produit par le logiciel CFD.
- Lancer le calcul PARK...

9.1 Pré-traitement

9.1.0 Génération des données pour les logiciels CFD

L'onglet *Pré-traitement* présenté à la Figure 1 apparaît après avoir lancé le calcul CFD-Interface et sélectionné l'option *Pré-traitement* dans l'onglet *Principal*.

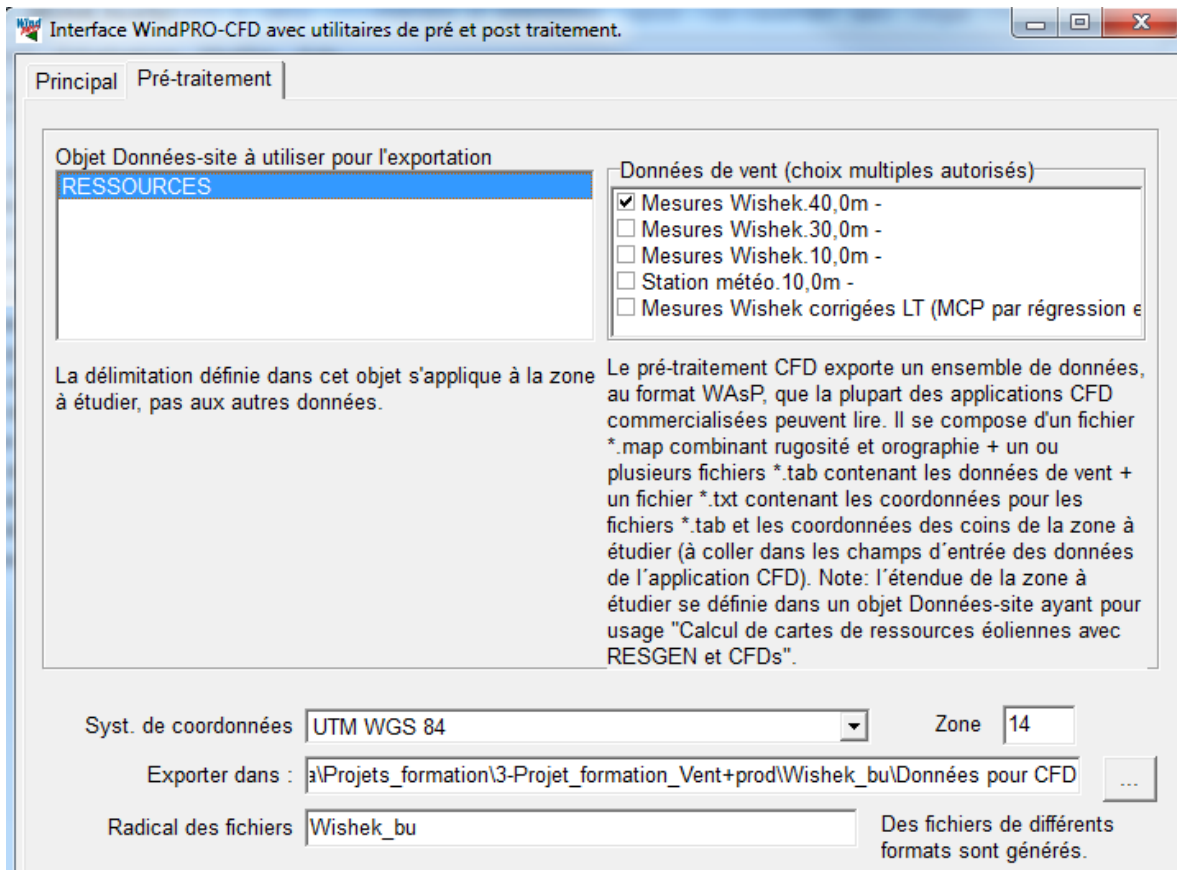


Figure 1

Les données d'orographie et de rugosité à « exporter » sont sélectionnées indirectement via un objet *Données-site*. Si plusieurs objets ont été créés dans le projet il faudra choisir l'objet idoine dans la liste *Objet Données-site à utiliser pour l'exportation*. Les données d'orographie et de rugosité seront exportées dans un fichier .map (format WASP) que la plupart des logiciels CFD savent lire. De plus, un fichier .txt contenant les informations de géoréférencement est également produit.

Les données de vent contenues dans un objet *Meteo* peuvent également être exportées dans un fichier au format .tab (WASP). Un fichier .txt où sont indiquées les coordonnées de l'objet *Meteo* et la hauteur des données de vent est également produit.

Le bouton à droite du champ *Exporter dans* permet de définir le dossier où seront enregistrés les fichiers décrits antérieurement. C'est ce dossier qui devra être transmis à l'exploitant du logiciel CFD.

9.2 Post-traitement

Le *Post-traitement* permet de comparer les résultats obtenus à partir du logiciel CFD avec ceux obtenus à partir de WindPRO et WAsP.

Pour le *Post-traitement* il est nécessaire que le logiciel CFD produise un fichier carte des ressources au format .rsf ou .wrg.

Etapes à suivre :

- Après avoir lancé le calcul CFD-Interface et sélectionné l'option *Post-traitement* dans l'onglet *Principal* apparaît l'onglet *Post-traitement* présenté à la Figure 2.
- Pour faire la comparaison des résultats il faut charger le fichier calculé avec WindPRO et WAsP et celui calculé avec le CFD en cliquant sur les boutons à droite des champs *1er fichier* et *2ème fichier*.
- Choisir la grandeur à comparer en la cochant dans le cadre *Afficher/comparer*.

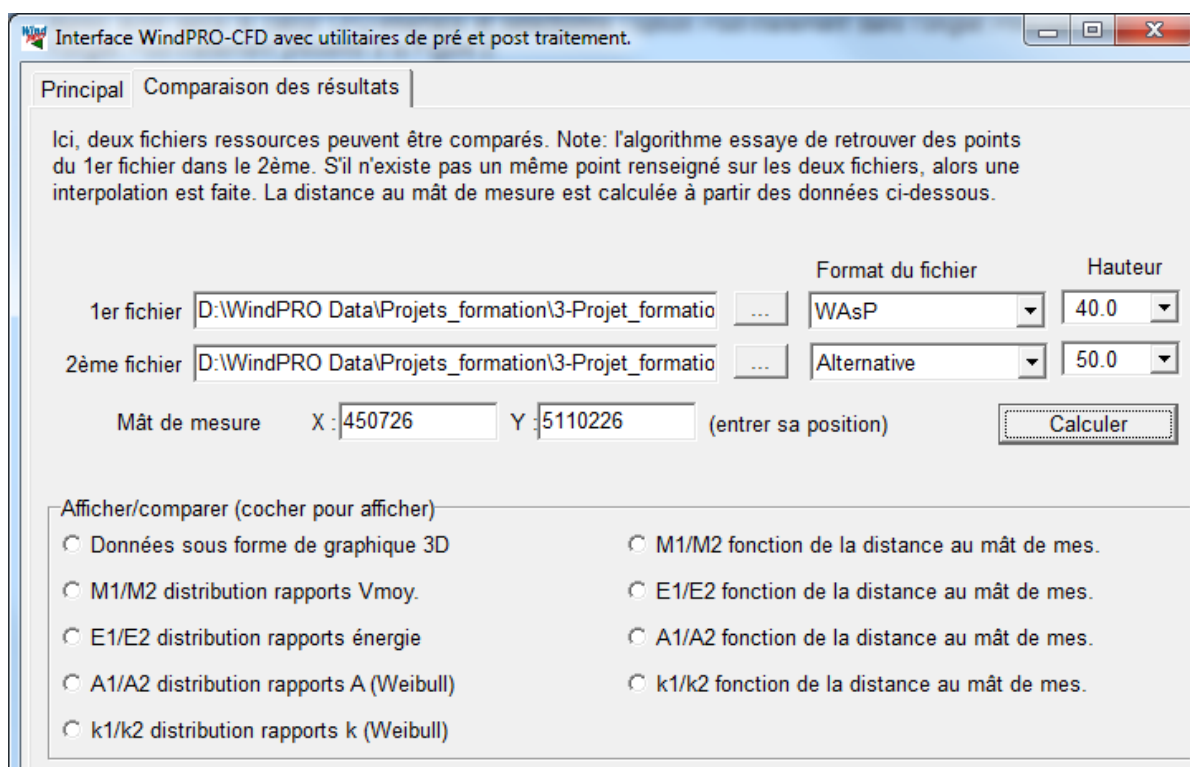


Figure 2

9.2.0 Compléments sur la comparaison des résultats

Les conditions de vent en chaque point de la carte sont données par les paramètres de Weibull A et k de la distribution du vent.

Pour effectuer les comparaisons point à point les paramètres A et k sont extraits des fichiers cartes des ressources produits par WindPRO+WAsP et par le CFD. Les grandeurs suivantes peuvent alors être comparées :

- M1/M2 : comparaison des vitesses moyennes du vent.
A cet effet, les deux V_{moy} en chaque point de la carte doivent être calculées à partir des paramètres A et k en utilisant la relation suivante :

$$V_{moy} = A \cdot \Gamma(1+1/k) \text{ où } \Gamma = \text{fonction Gamma}$$

Puis le rapport suivant est calculé pour chaque point :

$$R = \frac{V_{\text{moy}}[\text{CFD}]}{V_{\text{moy}}[\text{WAsP}]} = \frac{A[\text{CFD}] \cdot \Gamma(1 + 1/k[\text{CFD}])}{A[\text{WAsP}] \cdot \Gamma(1 + 1/k[\text{WAsP}])}$$

où $A[\text{WAsP}]$ et $k[\text{WAsP}]$ sont les paramètres A et k de la distribution de Weibull ajustée par WAsP pour le point considéré et $A[\text{CFD}]$ et $k[\text{CFD}]$ sont les paramètres A et k de la distribution de Weibull ajustée par le CFD pour le même point de la carte.

Note : WAsP ajuste la distribution de Weibull (A et k) à la distribution des vitesses de vent relevées de la manière suivante :

- l'énergie éolienne est identique à celle de la distribution des vitesses de vent relevées,
- la fréquence des vitesses supérieures à la V_{moy} des vitesses relevées est identique à celle de la distribution des vitesses de vent relevées.

Dans ces conditions la vitesse moyenne du vent calculée à partir des paramètres A et k déterminés par WAsP peut être sensiblement différente de la vitesse moyenne relevée, par conséquent il est préférable de comparer les rapports de l'énergie éolienne $E1/E2$.

- $E1/E2$: comparaison des énergies
- $A1/A2$: comparaison des paramètres A
- $k1/k2$: comparaison des paramètres k

Les résultats des comparaisons peuvent être présentés des manières suivantes :

- la simple distribution des rapports,
- les rapports en fonction de l'éloignement par rapport au *Mât de mesure*,
- représentation des rapports par des couleurs venant draper la représentation 3D filaire du relief de la zone étudiée.

9.3 Exemple. Etude du cas Torrild

9.3.0 Site and Analysis Conditions

A total of 15 Bonus 150/30 kW MK II turbines are situated on site. The power production from these turbines is used for verification purposes. The hub height is 30 meters.

Two models were run. The 'Meso'-model was run with a grid resolution of 200 meters resulting in 10000 nodes. The micro model has been nested within the Meso model. In the 'Micro' model test case, the grid resolution was chosen to be 20 meter, also resulting in 10000 nodes. Here, only results from the micro model will be presented. The main characteristics of the area are shown below:

Company	BONUS
Type/Version	MK II
Rated power	150.0 kW
Secondary generator	30.0 kW
Rotor diameter	23.8 kW
Tower	Tripod
Origin country	DK
Blade type	LM 11
Generator type	Two generator
Rpm, rated power	40.4 rpm
Rpm, initial	30.3 rpm
Default hub height	30.0 m
Alternativ hub heights	0.0 m
Valid	No
Creator	EMD
Created	07-10-1998 00:00
Edited	07-10-1998 00:00



Calculation area	4 km ²
Coordinate system	UTM zone 32 – datum ED50
Geographical limits	(y _{min} , y _{max})=(6203500, 6205500) (x _{min} , x _{max})=(563700, 565700)
Grid size	20 meters
Number of cells	10000
Wind data	Station 5 – Feb to Oct 2000 Station 36 – Feb to Oct 2000 (St 36: data acquisition very low)

A through description of the site with an analysis of the wind climate and the production from the turbines using traditional methods. The orography and roughness for the calculation area are shown in Figure 3. These figures represent the initial extraction area, which was 4 x 4 km². This was later reduced into a 2 x 2 km² area. The approximate position of the final calculation area is shown by the black square in the figures.

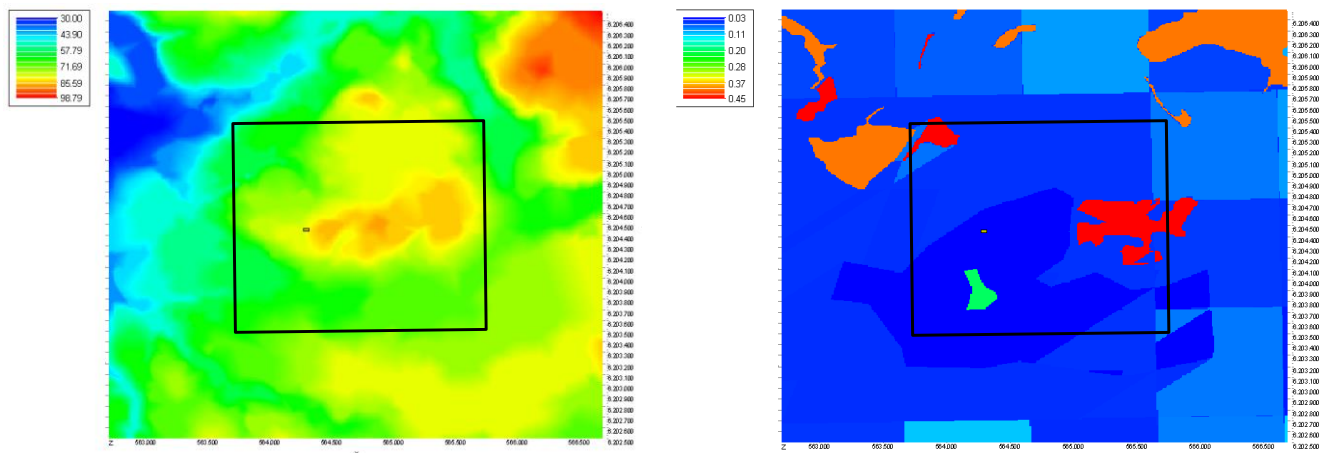


Figure 3: Orography and roughness from the initial extraction (orography in [m], roughness in [m]).

9.3.1 Comparing WAsP and WindSim CFD Results

The main statistics of the ratio between the WindSim and WAsP results are shown in Table 1. The mean value of the ratio is 97.2%, which mean that the WindSim calculations in general seem to underestimate the mean value of the wind speed distribution. However, this difference may also be due to differences in fitting techniques when fitting the Weibull parameters. The coefficient of variation is 2.4%, which is judged acceptable. When looking at the energy level, the WindSim calculation underestimates approximately – in mean - 9 % as compared to WAsP.

Mean wind speed			Energy level		
μ	σ	$COV=\sigma / \mu$	μ	σ	$COV=\sigma / \mu$
0.972	0.023	0.024	0.907	0.069	0.076

Table 1: Statistics on the ratio, R, between the mean wind speed or energy level calculated from WindSIM and WAsP.

The dependency on the distance from the meteorological mast and the height above ground is shown in Figure 4. It is obvious, that the WindSim model seems to underestimate increasingly as the distance from the meteorological mast increases. A larger variation is seen as in a distance approximately 1000 meters from the mast. Inspecting the Figure 5 and comparing with the roughness map in Figure 3 find the reason for this larger variation. Here it is seen, that the large ratio comes where the roughness is high (a small city is situated). Near the site, the two models predict quite similar, and the difference in mean wind speed is within a few per cent.

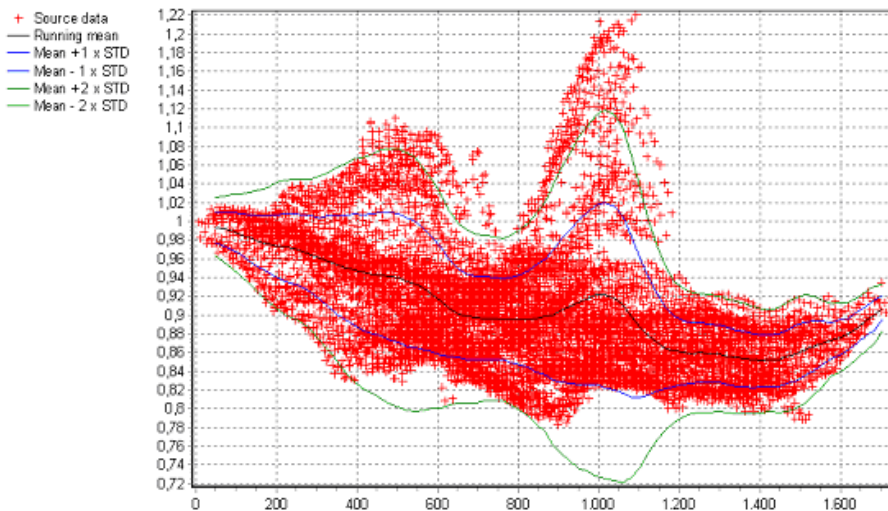


Figure 4: WindSim/ WAsP mean wind speed ratio conditioned on the distance from the met mast.

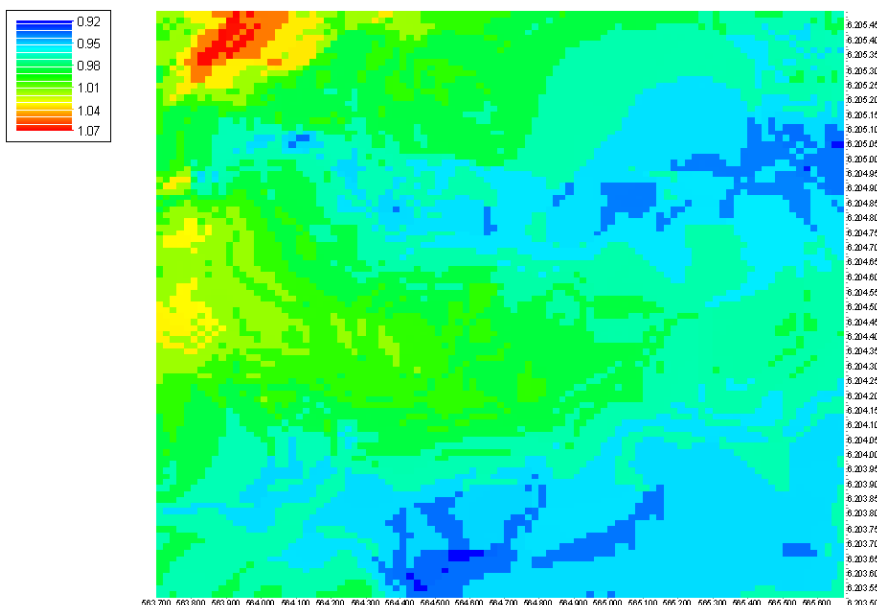


Figure 5: Comparing WindSim / WAsP mean wind speed ratio distributed over the calculation area.

9.3.2 Comparing Results at the Meteorological Mast

In order to compare how the different models perform, it is sought to estimate the wind climate at the position of the meteorological mast. I.e. it is possible to compare the model predictions with the original measured data. WAsP v. 5.1 is used to fit the Weibull parameters in all models in order to overcome the known problems of using different Weibull fitting algorithms (see Fitting Weibull Parameters for Wind Energy Applications).

Calculated Weibull A-parameters [m/s]

Model	Height	N	NNE	ENE	E	ESE	SSE	S	SSW	WSW	W	WNW	NNW	All
Measured	30 m	4.70	4.50	5.10	6.60	6.70	6.00	6.60	6.80	7.50	6.30	4.10	5.10	6.30
WAsP	30 m	4.73	4.67	5.15	6.59	6.73	6.03	6.57	6.90	7.42	6.35	4.59	5.11	6.34
	70 m	5.59	5.45	6.25	8.26	8.19	7.10	7.78	8.07	8.81	7.79	5.62	6.26	7.65
	100 m	6.08	5.93	6.82	8.95	8.79	7.70	8.48	8.72	9.52	8.43	6.08	6.79	8.28
WindSim	30 m	4.60	4.50	5.10	6.60	6.70	6.00	6.50	6.80	7.50	6.30	4.10	5.10	6.30
	70 m	5.30	5.20	6.00	7.60	7.60	6.60	7.30	7.50	8.30	7.10	4.90	5.90	7.10
	100 m	5.60	5.50	6.20	8.00	8.00	7.00	7.60	7.90	8.60	7.40	5.20	6.20	7.50

Calculated Weibull k-parameters

Model	Height	N	NNE	ENE	E	ESE	SSE	S	SSW	WSW	W	WNW	NNW	All
Measured	30 m	1.64	2.04	2.20	2.42	2.53	2.19	1.80	2.19	2.27	2.00	1.54	1.62	2.04
WAsP	30 m	1.72	2.08	2.19	2.44	2.53	2.19	1.87	2.21	2.28	2.04	1.63	1.65	2.06
	70 m	1.94	2.33	2.47	2.72	2.83	2.42	2.06	2.49	2.55	2.31	1.82	1.86	2.30
	100 m	2.02	2.42	2.54	2.80	2.92	2.50	2.15	2.59	2.67	2.42	1.89	1.94	2.30
WindSim	30 m	1.63	2.03	2.19	2.42	2.54	2.17	1.79	2.20	2.28	2.00	1.53	1.61	2.05
	70 m	1.63	2.06	2.24	2.44	2.57	2.14	1.81	2.19	2.30	2.02	1.59	1.63	2.07
	100 m	1.65	2.09	2.18	2.35	2.46	2.19	1.81	2.20	2.21	2.00	1.59	1.61	2.05

Calculated direction probability [%]

Model	Height	N	NNE	ENE	E	ESE	SSE	S	SSW	WSW	W	WNW	NNW	All
Measured	30 m	2.00	3.50	8.10	10.6	9.60	8.20	8.40	12.3	15.6	14.9	4.20	2.70	100
WAsP	30 m	2.10	3.80	7.80	10.5	9.70	8.30	8.60	12.4	15.5	14.4	4.60	2.70	100
	70 m	2.00	3.50	7.90	10.8	10.0	8.20	8.20	12.0	15.7	14.7	4.60	2.60	100
	100 m	2.00	3.50	7.90	10.9	9.90	8.10	8.20	12.0	15.7	14.7	4.50	2.60	100
WindSim	30 m	2.00	3.50	8.10	10.6	9.60	8.20	8.40	12.3	15.6	14.9	4.20	2.70	100
	70 m	2.00	3.50	7.90	10.8	9.70	8.00	8.40	12.2	15.5	15.1	4.20	2.60	100
	100 m	2.00	3.50	7.90	10.9	9.70	8.00	8.40	12.1	15.5	15.2	4.20	2.60	100

When comparing the measured and modelled Weibull A-parameters, it seems like the WAsP model has a tendency to slightly over predict the A parameter at the 30 meters height. The WindSim model gives a result that is very close to the measured data (as it is supposed to as the speedups and direction changes are 0). When looking at the A-parameter at 70 meters and 100 meters, then the WindSim model predicts a lower A-parameter than the WAsP model. There are actual plans on substituting the 15 x 150 kW WTGs with a few 2 MW WTGs – if this happen, we will get the chance to see which model that perform best in this matter.

In this test case, the modelled directional probabilities are not very different from the measured ones.

9.3.3 Estimating Meteorological Station 5 with Meteorological Station 36 data and vice versa

The measured data from station 5 and station 36 has been compared with WindSim modelled data at the same positions but using wind data from the other station. Unfortunately, the data from station 36 is erroneous in long periods, leaving us with only 17 days of good data from February 2000. The analysis of these remaining data is shown below. A WAsP analysis has been made for comparison.

In the Figure 6 the speedup data from the two stations are shown. The actual WindSim data are calculated at one wind speed only (approx. 7-8 m/s), and the data is assumed to have a linear development as shown on the figures – passing through 0.0. A speedup-development is assumed to be stepwise linear if more calculation wind speeds are added. Examining the two figures it is easy to see, that at 10 m/s the maximum speed-up is approximately 0.3 m/s, i.e. a change no more than 3%. This means that we should expect only small changes in the modified wind distributions.

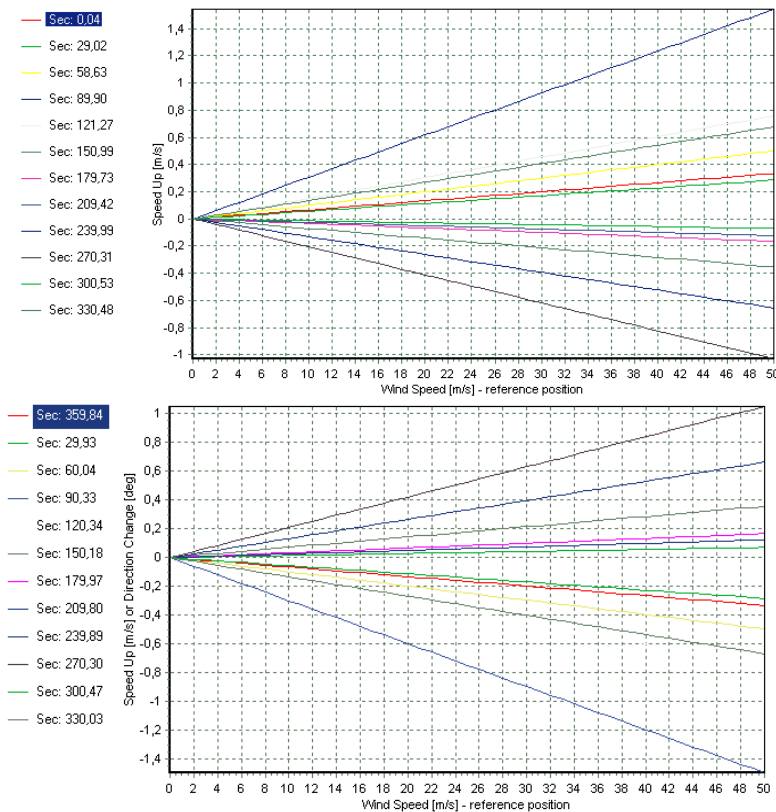


Figure 6: Speedup data from calculated sectors (left=St 36 using St 5, right = St 5 using data from St 36)

WindSim results

The measured distributions for the 17-day period and the estimates of the new wind and directional distributions are shown in Figure 7 and Figure 8. It is obvious that the predicted distributions at the other position looks very much like the one that they are derived from. This is due to the very small modification factors for all dominating sectors (SSW to NNW). It seems that our model has not been able to capture the large differences in the two measured distributions. However, when over viewing the site, then it is obvious that both of the meteorological masts are operating in wind turbine wakes for long periods. The meteorological mast number 36 is placed only 70 meters from the near most turbine. Making a park analysis in WindPRO the 'park efficiency' is found to 92.2% for station 5 and station 36 is at 87.7% (park efficiency is calculated relative to energy levels). Interpreting the park efficiency in terms of wind speeds, approximate calculations shows that the omni directional mean wind speed at the sites is lowered with 0.19 m/s (station 5) and 0.30 m/s (station 36) relative to the free wind speed. Thus, it is concluded, that the WindSim model performs as expected and the main difference in measured wind statistics is due to wind farm wake effects. In addition, differences in anemometer calibration and/or type may cause differences in wind statistics, but this issue is not investigated further here.

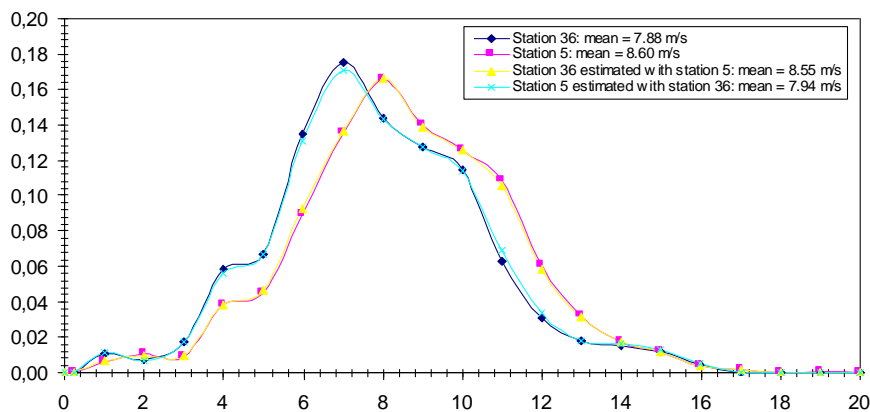


Figure 7: Sample and modified distributions for Stations 5 and 36.

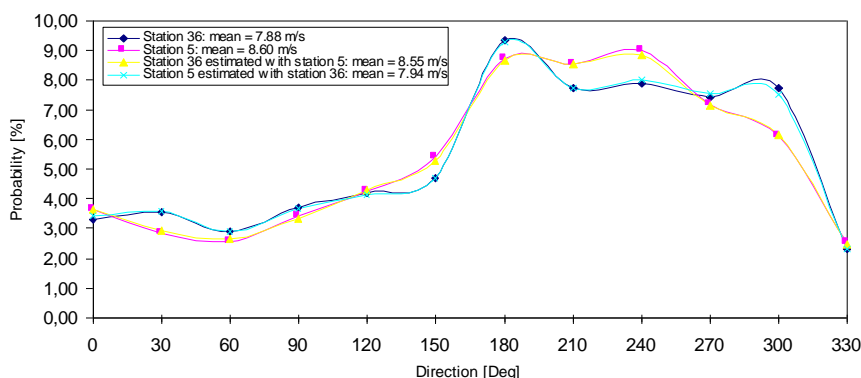


Figure 8: Directional distributions for stations 5 and 36 and the model data.

WASP results

In order to support the conclusions regarding the main differences in measured wind distributions a WASP analysis has been performed, analysing differences in the two meteorological stations by using the 'long term' data from station 5. Only the Weibull A-parameter is reported below in Table 2. It is seen, that the differences at the two sites are almost negligible, supporting the conclusion that the main differences in the measured wind statistics comes from the wake effects.

Station	N	NNE	ENE	E	ESE	SSE	S	SSW	WSW	W	WNW	NNW	All
St 5 measured (long)	4.70	4.50	5.10	6.60	6.70	6.00	6.60	6.80	7.50	6.30	4.10	5.10	6.30
St 5 with 5 long term	4.73	4.67	5.15	6.59	6.73	6.03	6.57	6.90	7.42	6.35	4.59	5.11	6.34
St 36 with 5 long term	4.68	4.53	5.10	6.75	6.84	5.98	6.50	6.83	7.67	6.74	4.68	5.17	6.45

Table 2: Weibull A-parameters for estimated with different meteorological stations (5 and 36).

9.3.4 Comparing with production data

The production data from the turbines have been analysed in [i], and the results from this reference is used here. The WindSim results have been save in a rsf-file and processed using the PARK module in WindPRO in order to include wake effects. The results from the analysis are shown in the Figure 9 where data from the 'Goodness' indexes for all turbines on site is plotted. The 'Goodness' is defined as the ratio between the actual measured production and the calculated (modelled) production, i.e. a 'Goodness' larger than 1.0 is an underestimation of the actual measured production.

When inspecting the Figure 9 it is obvious that both models seem to underestimate the actual measured production. As stated before in 3.3, the WAsP model over-estimates the wind climate even if the same position is estimated. This may account for some of the difference between the two models. Also, results from the WAsP model seems to somewhat correlate to the height (green line in graph below), i.e. 'high' altitudes may also be followed by a relative large 'Goodness'. The WindSim model does not have the same trend, but seems to increase in 'Goodness' as the distance from the meteorological mast increases.

So a promising thing about WindSim is that is seem to "catch" the lower sited WTGs much better (WTG 5 and 9), which indeed is a positive trend. Thus, this may invoke much improvement at sites with more complex terrain. WindSim predicts the WTGs farther away from met mast worse. This may be caused by the fact that a WindSim micro siting analysis does not take roughness at longer distance into the calculation (except in cases where the analysis has been run as a nested analysis). In general, it seems that the 'physical' roughness description used in WindSim does not capture the effect of the complex roughness as well as the more refined (but experience based) treatment of roughness in WAsP. This lead to conclusion, that WindSim performs its optimum at sites with more simple roughness and complex orography.

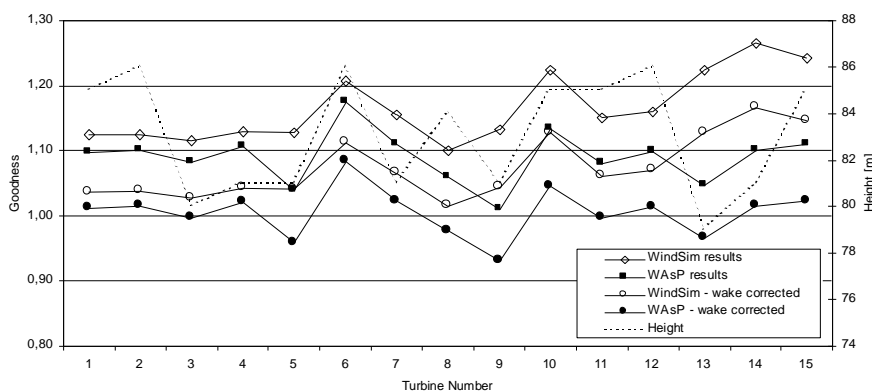


Figure 9: Comparing energy production calculated from WindSim with those calculated with WAsP.

When finding that both initial WindSim and WAsP calculation as well underestimate the actual measured production, then we may look for other sources of error than pure model errors. This may be a too conservative power curve, however the power curve seems fine when compared to the HP generic power curves, see [i]. Another obvious source of error could be wind speeds measured too low. This may be initiated by:

- Boom effects
- Wake effects from the nearby turbines (which may be the most probable cause)
- Anemometer errors (e.g. from wear and tear) – calibration required
- False positioning of met mast relative to height contours (or 'disturbances' in the actual wind profile – e.g. a zero displacement height)

As stated in Section 3.3.4, the wakes from nearby turbines influences the measured wind speeds significantly. When making a rough correction of the wake influence (simply by adjusting the calculated wind energy with the 'park efficiency' factors as stated in Section 3.3.4), then the WAsP results are now within $\pm 9\%$. The WindSim calculations still – in mean - underestimate the actual production, but within a reasonable margin, see Figure 9 and Table 3. One main reason for the underestimation of the energy is believed to rise from lack of a good/validated free wind distribution. In new wind farm projects, this issue does not occur, but in validation cases, it is of utmost importance to have tools for analysing and 'cleaning' the data from wakes. This feature is currently being implemented in an upcoming version of WindPRO.

Statistic	Goodness from	
	WindSim	WAsP
Max	1.167	1.083
Min	1.014	0.931
Mean	1.075	1.005
Standard deviation	0.048	0.037

Table 3: Goodness - Wake Corrected.

9.3.5 Discussion/Conclusion of case study

The case study has demonstrated the application of the new tools available in WindPRO for linking existing data into a WindSim/WindPRO joint analysis. The ease of extracting the WindSim input files enables the user to gain the benefit of a second opinion analysis.

The actual case study from Torrild shows, that the site fits the WAsP model best – it is a typical Danish site – even if the orography is some of the ‘roughest’ that we may find in Denmark. The WindSim model seems to have problems with taking complex roughness into account, but it also seem to improve handling of the orography. In addition, it must be noted that the WindSim model has its strengths in sites with complex orography and not sites with complex roughness - as the current site.

9.5 Appendix. Making Joint Wind Distribution Modifications

9.5.0 On-Site Wind Distributions

Traditionally in wind resource assessment, the on-site (subscript: site) and reference-site (subscript: ref) wind speed distribution are given as discrete distributions, showing the distribution of 10-minute averaged wind speed conditioned on the different wind sectors. In addition, the distributions of the wind directions (sectors) are given:

$$P(\theta_{ref}) \text{ and } P(U_{ref} | \theta_{ref})$$

These two distributions are enough for establishing the joint distribution for the reference site:

$$P(U_{ref}, \theta_{ref}) = P(U_{ref} | \theta_{ref}) \cdot P(\theta_{ref})$$

The joint distribution of (U_{ref}, θ_{ref}) is easily extracted from the *.tab file as shown in the Figure 10.

9.5.1 New wind distribution using a JPC-method (JPC=Joint Probability Change)

The distribution of wind speeds and wind directions are given as a discrete joint distribution, e.g. specified by a *.TAB file. The results from WindSim are assumed to be given as node specific speed up addenda, S , and direction change addenda, D , which both are – in the general case – functions of the wind speed and direction on the reference site $S=f(U_{ref}, \theta_{ref})$, $D=g(U_{ref}, \theta_{ref})$. It is now assumed, that the speed up factors and directions are reported in a table with the results is extracted for each calculation node. The reference site samples are - within one bin of wind speed and direction - assumed to follow a joint uniform distribution. The density function is given by:

$$f(u, \theta) = \begin{cases} \frac{1}{\beta - \alpha} \cdot \frac{1}{\delta - \gamma} & \text{if } (\alpha \leq u \leq \beta) \wedge (\gamma \leq \theta \leq \delta) \\ 0 & \text{otherwise} \end{cases}$$

where u is the wind speed at the reference site in the bin considered
 θ is the wind direction at the reference site in the bin considered
 α and β are the lower and upper limits for the wind speed interval
 γ and δ are the lower and upper limits for the wind direction interval

The probability that (u, θ) lies in a interval between $[a, b]$ and $[g, d]$ is found by:

$$P(a < u < b, g < \theta < d) = \frac{b-a}{\beta-\alpha} \cdot \frac{d-g}{\delta-\gamma} \quad \text{where} \quad \begin{cases} a = \alpha & \text{if } a < \alpha \\ a = \beta & \text{if } a > \beta \\ b = \alpha & \text{if } b < \alpha \\ b = \beta & \text{if } b > \beta \\ g = \gamma & \text{if } g < \gamma \\ g = \delta & \text{if } g > \delta \\ d = \gamma & \text{if } d < \gamma \\ d = \delta & \text{if } d > \delta \end{cases}$$

Now the wind speed from the reference site, u_{br} , is added with the speed up addendum, S , and the wind direction from the reference site, θ_{br} , are added with the direction change, D . This yields two new random variables, describing the wind climate at the site.

$$u_{bs} = u_{br} + S$$

$$\theta_{bs} = \theta_{br} + D$$

Now it is possible to evaluate the new 'bin density function' from the old one by adding the (S, D) to the limits of the original bin joint distribution. Now the probability of being within the bin-limits of the new joint distribution is easily evaluated, i.e. creating a new wind and direction joint distribution by moving the probability mass by (S, D) .

Example Torrild – Cell 3569

The probability of being in the east-sector is $P[\theta_{br} = 75^\circ - 105^\circ] = 0.106$. The probability of being in the bin between 7 m/s and 8 m/s given that the wind is coming from the east sector is $P[u_{br} = 7 - 8 \text{ m/s} | \theta_{br} = 75^\circ - 105^\circ] = 0.147$. Now, the joint probability of being in the bin 7-8 m/s and having wind from east is:

$$P[u_{br} = 7 - 8 \text{ m/s}, \theta_{br} = 75^\circ - 105^\circ] = P[u_{br} = 7 - 8 \text{ m/s} | \theta_{br} = 75^\circ - 105^\circ] \cdot P[\theta_{br} = 75^\circ - 105^\circ]$$

$$= 0.015582$$

i.e. approximately 1.5% chance of being in that particular bin at the reference position.

Inspecting the equation above describing the joint uniform distribution, then $\alpha = 7 \text{ m/s}$, $\beta = 8 \text{ m/s}$, $\delta = 75^\circ$ and $\gamma = 105^\circ$. Taking cell number 3569 at position E:568400, N: 6201600 from the WindSIM result files, then the speedup may be found for the particular bin to $S = -0.704 \text{ m/s}$ and the direction change is $D = 0.118^\circ$, making it possible to calculate the new limits for the old 'bin' probability mass, $\alpha_{\text{new}} = 6.296 \text{ m/s}$, $\beta_{\text{new}} = 7.296 \text{ m/s}$, $\delta_{\text{new}} = 75.118^\circ$ and $\gamma_{\text{new}} = 105.118^\circ$.

Evaluating the above equation (considering all proper limitations) for the nearby bins, one obtains the results reported in the table 4 below. The table is showing how much of the bin-probability (0.015582) that moves into other (nearby bins). It is seen, that the main change is within the sector as the direction change is very small. Only 29% of the winds remains in the same bin. This procedure is repeated for all bins, calculating a new joint distribution for the site-position.

	$\theta_{bs} =$ $45^\circ - 75^\circ$	$\theta_{bs} =$ $75^\circ - 105^\circ$	$\theta_{bs} =$ $105^\circ - 135^\circ$	$\theta_{bs} =$ $135^\circ - 165^\circ$
$u_{bs} = 5 - 6 \text{ m/s}$	0.00000	0.00000	0.00000	0.00000
$u_{bs} = 6 - 7 \text{ m/s}$	0.00000	0.70123	0.00277	0.00000
$u_{bs} = 7 - 8 \text{ m/s}$	0.00000	0.29484	0.00116	0.00000
$u_{bs} = 8 - 9 \text{ m/s}$	0.00000	0.00000	0.00000	0.00000

Table 4: Example Torrild: Probability Mass Moved Into Nearby Bins.

The original joint wind and direction distribution are given in Figure 10, while Figure 11 and Figure 12 show the speedup and direction changes respectively. Speedups are interpolated and extrapolated linearly while the direction changes are linearly interpolated but constant when extrapolated (or only one calculation wind speed).

Torrild Meteorological Site Data (Sample)
 55.979786111 10.03035 30.00
 12 1 0

	0.02	0.035	0.081	0.106	0.096	0.082	0.084	0.123	0.156	0.149	0.042	0.027
1	48	20	10	8	12	6	4	12	7	10	23	59
2	97	71	59	74	30	29	20	29	24	61	85	153
3	168	170	111	115	82	125	63	34	29	111	218	165
4	175	234	179	142	121	139	88	50	60	138	170	216
5	133	163	187	151	235	156	93	97	87	122	96	89
6	92	123	146	143	176	107	117	106	91	109	95	122
7	46	78	136	142	175	86	124	117	118	77	79	41
8	30	51	94	147	116	98	92	136	137	99	94	31
9	97	43	57	61	32	64	84	109	94	94	60	38
10	76	36	14	15	17	48	68	89	83	74	44	46
11	37	10	4	3	3	42	56	101	70	49	18	36
12	2	1	1	0	0	42	39	66	67	21	6	5
13	0	0	0	0	0	34	50	27	56	15	1	0
14	0	0	0	0	0	17	58	17	28	11	6	0
15	0	0	0	0	0	5	30	5	16	4	4	0
16	0	0	0	0	0	1	12	2	12	3	1	0
17	0	0	0	0	0	0	2	2	12	2	0	0
18	0	0	0	0	0	0	1	6	0	0	0	0
19	0	0	0	0	0	0	0	1	2	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0

Figure 10: Original Torrild Joint Wind and Direction Distribution (reference position).

	Sec: 0	Sec: 30	Sec: 60	Sec: 90	Sec:120	Sec:150	Sec:180	Sec:210	Sec:240	Sec:270	Sec:300	Sec:330
0	0	0	0	0	0	0	0	0	0	0	0	0
20	-1.141	-1.183	-1.801	-1.76	-1.991	-2.276	-1.946	-1.528	-1.126	-1.414	-1.417	-1.279

Figure 11: Direction Change Data for Torrild Cell 3569 (the direction change is assumed fixed for all bins)

	Sec: 0	Sec: 30	Sec: 60	Sec: 90	Sec:120	Sec:150	Sec:180	Sec:210	Sec:240	Sec:270	Sec:300	Sec:330
20	-0.299	-0.074	0.074	0.118	0.133	0.327	-0.322	-0.37	-0.026	0.341	0.301	0.037

Figure 12: Speedup-data for Torrild Cell 3569 (the data is linear- interpolated for bins between 0-20 m/s Reference)

Reference

[i] Per Nielsen et al.: *Case Studies: Wind Energy Calculation Methods and Verification (preliminary title)*, Energi- og Miljødata, 2002.